Robust Design

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For the modern industry practice, an excellent design of any product aims to shift resources to the creative design process rather than relying on inspection to ensure quality. A quality characteristic is identified, and quality is achieved by minimizing deviation from its target rather than mere conformance to specification. To achieve these purposes, a robust design is then defined as

A Design is robust when the product performance is minimum sensitive to:

- **Material Variation**
- **Manufacturing Variation**
- **Operation Variation**

The Robust Design can be illustrated by the following figure.

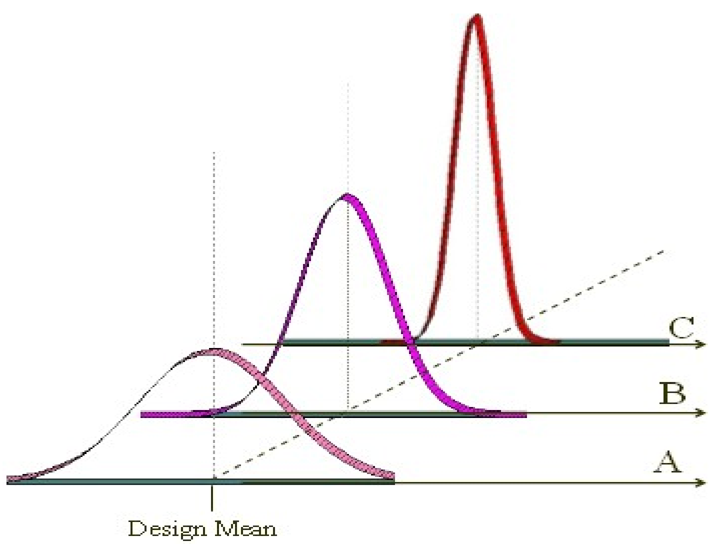


Figure 1 Illustration of robust design

In this figure, all three designs have the same mean value. However, Design is the best design because it has the least deviation from the mean. A robust design is the one has the minimum standard deviation in the entire design space with the same design mean. The minimal variation means the highest reliability. This is illustrated in the following figure.

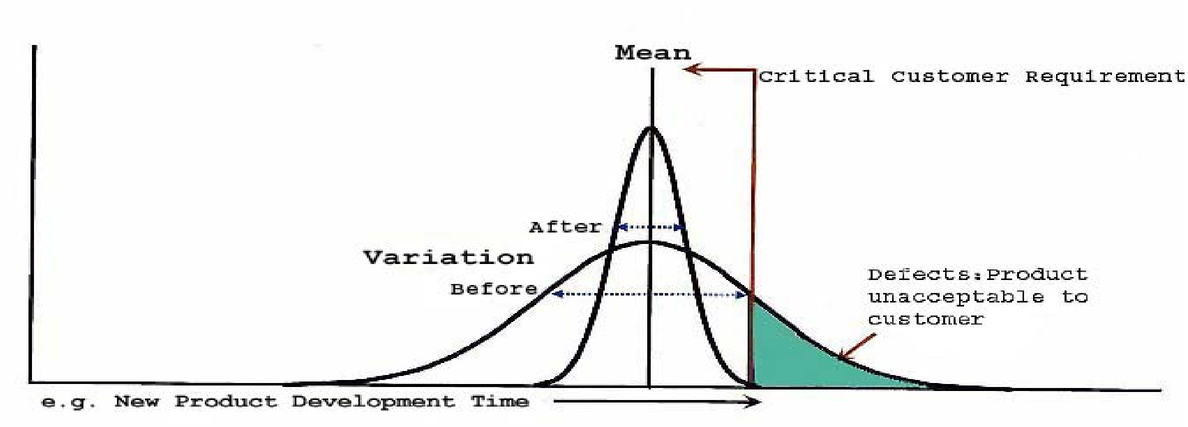


Figure 2 The Robust Design reaches the highest reliability

Example 1: Robust Design for a Simple Cylinder with normal distributed design variables

A simple cylinder as shown in the following figure is used to demon the Robust Design. This cylinder is designed as a constant volume and the standard deviations of design variables D (diameter) and H (height) are assumed to be constant value of 05. What are the robust Design of D and H if the volume of cylinder is held as 311?

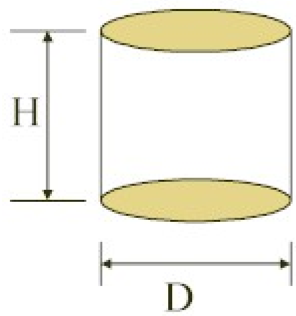


Figure 3 ***Robust Design of a Simple Cylinder***

Solution:
The design variables have constant standard deviation of 0.5 and their means are random variables (Hyper parameter distributions). It is noted that that the design variables re always entered after the variables of range. The random variables are then set up as the first two random variables are the range of design variables and the two design variables are hyper parameter distribution as shown below.

Variables : x1 = D Range: Uniform(4, 12)
x2 = H Range: Uniform(4, 12)
x3 = D: Normal(x1, 0.5)
x4 = H: Normal(x2, 0.5)

The volume of the simple cylinder is held as constant of 311. therefore, the limit state function is defined as following equation.

Limit State Function: $g(\underline{x}) = 311 - \frac{\pi}{4} \cdot x_3^2 \cdot x_4$

The final result is summarized as below:

******* ROBUST DESIGN *******

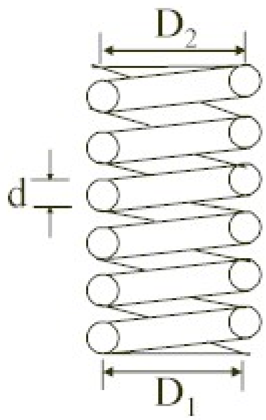
```
1st-order standard dev. of g(x).......= 4.6235356E+01
```

value of limit-state function.....g(x)= 3.07430E-05

Design Variables	Robust Point
width	8.37684E+00
height	5.64300E+00

Example2: Robust Design for the design of the spring using Helical Spring Design Equation

A spring stiffness expressed by Helical Spring Design Equation is used to as shown in the following figure is used to demon the Robust Design. The Helical Spring Equation is given in the following equation.



$$k = \frac{G * d^4}{2 * N * \left(D_1^2 + D_2^2\right) * \left(D_1 + D_2\right)}$$

Figure 4. Robust Design of a Simple Spring

where the design parameters are have the statistical data shown in the following table.

Parameter	Description	Distribution	Mean	Std. Dev.
d	Wire Diameter	Normal	0.125	0.00104
D	Coil diameter	Normal	2.0	0.028
G	Modulus	Normal	1.5E6	0.346E6
N	Active number of Coils	Deterministic	10	0

The spring stiffness *K* is equal to 4 at the mean point of the above table. The robust design for this spring is set sup the spring stiffness *K* as a constant value of 4 and keep the standard deviations of the design variables unchanged as in the above table.

Solution:

This is 4 dimensional robust design problem with the desired response of 4. The program can be illustrated as the following program diagram:

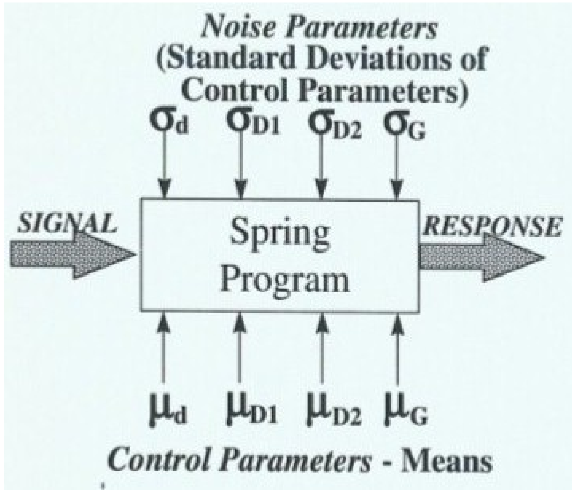


Figure 5 Program Diagram

In order to use UNIPASS to identify the Robust Point, the random variables are set up as the following Table:

Variables :

Random Variables	**Description**	**Distribution**	**Lower Bound**	**Upper Bound**
X1	Range of Wire dia.	Uniform	0.05	0.25
X2	Range of Coil dia. 1	Uniform	1	3
X3	Range of Coil dia. 2	Uniform	1	3
X4	Range of Modulus	Uniform	0.9E7	1.3E7
Random Variables	**Description**	**Distribution**	**Mean**	**Stand. Dev.**
X5	iWre dia., d	Normal	X1	0.00104
X6	Coil dia., D1	Normal	X2	1 0.028
X7	Coil dia., D2	Normal	X3	2 0.028
X8	Modulus	Normal	X4	0.346E6

Limit State Function:

$$g(x) = 4 - \frac{X_8 * X_5{}^4}{2 * 10 * \left(X_6^2 + X_7^2\right) * \left(X_6 + X_7\right)}$$

The final result is summarized as below

***** ROBUST DESIGN *****

```
1st-order standard dev. of g(x).......= 2.0727449E-01
value of limit-state function.....g(x)= 1.22791E-08
```

Design Variable	Robust Point
X5	1.38422E-01
X6	1.50875E+00
X7	2.91028E+00
X8	1.03478E+07